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Non-extremal gravity duals for fractional D3-branes on the conifold

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Abstract: The world volume theory on N regular and M fractional D3-branes at the conifold singularity is a non-conformal $\mathcal{N}=1$ supersymmetric $\mathrm{SU}(N+M)\times\mathrm{SU}(N)$ gauge theory. In previous work the extremal Type IIB supergravity dual of this theory at zero temperature was constructed. Regularity of the solution requires a deformation of the conifold: this is a reflection of the chiral symmetry breaking. To study the non-zero temperature generalizations non-extremal solutions have to be considered, and in the high temperature phase the chiral symmetry is expected to be restored. Such a solution is expected to have a regular Schwarzschild horizon. We construct an ansatz necessary to study such non-extremal solutions and show that the simplest possible solution has a singular horizon. We derive the system of second order equations in the radial variable whose solutions may have regular horizons.

Keywords: Duality in Gauge Field Theories, D-branes, Black Holes in String Theory, AdS-CFT Correspondance.

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Contents

1. Introduction

In this paper we study non-extremal generalizations of the KT solution [1], which describes regular and fractional D3-branes at the apex of the conifold. The extremal KT solution is crucially dependent on the presence of Chern-Simons terms in type IIB supergravity. These terms cause the RR 5-from flux to vary radially. Indeed, while in regular D3-brane solutions $dF_5 = 0$, here the 3-form field strengths are turned on in such a way that the right-hand side of the equation

$$dF_5 = H_3 \wedge F_3 \tag{1.1}$$

does not vanish. In fact, the 5-form flux increases without bound for large r. In [2] this behavior was attributed to a cascade of Seiberg dualities in the dual $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(N) \times \mathrm{SU}(N + M)$ gauge theory.

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Further examples of supergravity backgrounds with varying flux were constructed in [2]–[5]. In particular, it is important to understand the resolution of the naked singularity present in the KT solution. The proposal of [2] is that the conifold becomes deformed. As a result of this deformation the extremal KS solution is perfectly non-singular and without a horizon in the IR, while it asymptotically approaches the KT solution in the UV (for large ρ). The mechanism that removes the naked singularity is related to the breaking of the chiral symmetry in the dual $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ gauge theory. The Z_{2M} chiral symmetry, which may be approximated by U(1) for large M, is broken to Z_2 by the deformation of the conifold [2].

In [6] a different mechanism for resolving this naked singularity was proposed. It was suggested that a non-extremal generalization of the KT solution may have a regular Schwarzschild horizon "cloaking" the naked singularity. The dual field theory interpretation of this would be the restoration of chiral symmetry at a finite temperature T_c [6]. One expects that turning on finite temperature in the field theory, which translates into non-extremality on the supergravity side [7, 8], leads to restoration of the chiral symmetry above some critical temperature T_c.[1] The symmetry restoration is part of the deconfinement transition at T_c for this particular $\mathcal{N} = 1$ gauge theory. The proposal of [6] is that the description of the phase with restored symmetry involves a regular Schwarzschild horizon appearing in the asymptotically KT geometry. This proposal is analogous to the fact that the $\mathcal{N} = 4$ SYM theory, which is not confining, is described at a finite temperature by a black hole in AdS_5 [7, 8]. The difference is that in our case the $T = 0$ theory exhibits confinement and chiral symmetry breaking [2]. So, the regular Schwarzschild horizon should appear only for some finite Hawking temperature. This would be a rather unusual and, to our knowledge, unstudied phenomenon from the supergravity point of view.

One implication is that, at temperatures below T_c, there should be non-extremal generalizations of the KS solution which are free of horizons, just like the extremal solution. The absence of a horizon is a manifestation of confinement, as evidenced by the resulting area law for Wilson loops or, alternatively, by counting of degrees of freedom. Once the horizon appears, the Bekenstein-Hawking entropy associated with it typically scales as N^2 where N is the relevant number of colors. This factor indicates that the color degrees of freedom are not confined. If there is no horizon, then the entropy could only appear through string loop effects which would make it of order N^0, in agreement with color confinement. It is interesting, therefore, to study this theory as a function of the temperature, and to identify the phase transition where the chiral symmetry is restored.

[1]Strictly speaking, we cannot *a priori* rule out the possibility that $T_c = \infty$. In that case, a regular black hole in KT geometry does not exist. While this possibility seems strange, the only way to decide the issue is to find the actual black hole solution where a regular Schwarzschild horizon does shield the singularity.

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In order to address these questions from a dual supergravity point of view, we need to study non-extremal generalizations of the KT and KS backgrounds. This problem was first addressed for the KT background in [6] partly with numerical methods. We rederive this solution analytically and show that the identification of the horizon as $r = r_*$ in [6, eq. (2.60)] is incorrect because $\Delta_1(r_*) \neq 0$. The vanishing of $\Delta_2(r_*)$ is an artefact of the coordinate choice. We find a good radial coordinate u and show that the solution derived in [6] has a singular horizon at $u = \infty$ which corresponds to $r = \infty$ on another branch of the solution. This type of singular horizon is deemed unacceptable in studies of black hole metrics.

Thus, the solution found in [6] does not have a regular Schwarzschild horizon shielding the naked singularity of the extremal KT metric for sufficiently high Hawking temperature. Instead, the horizon (defined as the locus where $G_{00} = 0$) and the singularity are coincident, independent of the choice of the non-extremality parameter. We also show that, in the limit where we remove the fractional D3-branes (wrapped D5-branes), and leave only the regular D3-branes, the solution of [6] does not reduce to the standard non-extremal 3-brane metric. Instead, it reduces to a non-standard non-extremal version of a D3-brane solution whose metric has a singular horizon.

Nevertheless, the scenario for chiral symmetry restoration proposed in [6] is very appealing. This motivates us to introduce a more general U(1) symmetric ansatz and to begin search for solutions that are asymptotically KT but possess regular Schwarzschild horizons.

2. Non-extremal generalization of the KT ansatz

We start with an ansatz for the non-extremal KT background. Just as in [1] we impose the requirement that the background has a U(1) symmetry associated with the U(1) fiber of $T^{1,1}$. Our ansatz will be more general than that of [6]. It turns out that in order to look for solutions which reduce to the standard non-extremal D3-brane in the limit of zero fractional brane charge P, one should not impose self-duality on the 3-forms away from extremality. This in turn implies that one is to adopt a more general ansatz for the metric than in [6] and also allow for a non-constant dilaton.

A general ansatz for a 10-d Einstein-frame metric consistent with the U(1) symmetry of ψ-rotations and the interchange of the two S^2's involves 4 functions x, y, z, w of a radial coordinate u

$$ds^2_{10E} = e^{2z}(e^{-6x}dX_0^2 + e^{2x}dX_idX_i) + e^{-2z}ds_6^2\,, \tag{2.1}$$

where

$$ds_6^2 = e^{10y}du^2 + e^{2y}(dM_5)^2\,, \tag{2.2}$$

$$(dM_5)^2 = e^{-8w}e_\psi^2 + e^{2w}(e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2) \equiv e^{2w}ds_5^2\,, \tag{2.3}$$

and

$$e_\psi = \frac{1}{3}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)\,, \quad e_{\theta_i} = \frac{1}{\sqrt{6}}d\theta_i\,, \quad e_{\phi_i} = \frac{1}{\sqrt{6}}\sin\theta_i d\phi_i\,.$$

Here X_0 is the euclidean time and X_i are the 3 longitudinal 3-brane directions.

This metric can be brought into a more familiar D3-brane form

$$ds^2_{10E} = h^{-1/2}(\rho)[g(\rho)dX_0^2 + dX_i dX_i] + h^{1/2}(\rho)[\mathrm{g}^{-1}(\rho)d\rho^2 + \rho^2 ds_5^2]\,, \tag{2.4}$$

with the redefinitions

$$h = e^{-4z-4x}\,, \qquad \rho = e^{y+x+w}\,, \qquad g = e^{-8x}\,, \qquad e^{10y+2x}du^2 = \mathrm{g}^{-1}(\rho)d\rho^2\,. \tag{2.5}$$

When $w = 0$ and $e^{4y} = \rho^4 = 1/(4u)$, the transverse 6-d space is the standard conifold with $M_5 = T^{1,1}$. Small u thus corresponds to large distances (where we shall assume that $h, g, v \to 1$, as $\rho \to \infty$) and vice versa.

The function w squashes the U(1) fiber of $T^{1,1}$ relative to the 2-spheres; it does not violate the U(1) symmetry. A Ricci-flat 6-d space with non-trivial w is the generalized conifold of [9]

$$ds_6^2 = \kappa^{-1}(\rho)d\rho^2 + \rho^2[\kappa(\rho)e_\psi^2 + e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2]\,, \tag{2.6}$$

$$\kappa(\rho) = e^{-10w} = 1 - \frac{\rho_*^6}{\rho^6}\,, \qquad \rho = e^{y+w}\,, \qquad \rho_* \le \rho < \infty\,. \tag{2.7}$$

This space has regular curvature, and a bolt singularity at $\rho = \rho_*$ is removed by Z_2 identification of the angle ψ. The limit of the standard conifold is $\rho_* \to 0$ or $y_* \to -\infty$ which corresponds to $w = 0$.

The extremal D3-brane on the conifold and the more general fractional D3-brane KT solution have $x = w = 0$ (for their $w \neq 0$ analogs in the case when the 6-d space is the generalized conifold see [9]). Adding a non-constant $x(u)$ drives the non-extremality. For example, the non-extremal version of a D3-brane on a standard ($w = 0$) conifold solution has $x = au$, $e^{-8x} = g = 1 - 2a/\rho^4$, $e^{-4z-4x} = h = 1 + \tilde{q}/\rho^4$, $\rho = e^{y+x}$. Our aim will be to understand how switching on the non-extremality ($x = au$) changes the extremal KT solution.

Our ansatz for the p-form fields is dictated by symmetries and thus is exactly the same as in the extremal KT case [1]:[2]

$$F_3 = Pe_\psi \wedge (e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2})\,, \tag{2.8}$$

$$B_2 = f(u)(e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2})\,, \tag{2.9}$$

$$F_5 = \mathcal{F} + *\mathcal{F}\,, \qquad \mathcal{F} = K(u)e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} \wedge e_{\theta_2} \wedge e_{\phi_2}\,. \tag{2.10}$$

[2]The function T in [1] is related to f by $f = \frac{1}{\sqrt{2}}T$, and we make similar rescaling of P: $P = \frac{1}{\sqrt{2}}P_{KT}$.

As in [1], the Bianchi identity for the 5-form, $d * F_5 = dF_5 = H_3 \wedge F_3$, implies

$$K(u) = Q + 2Pf(u)\,. \tag{2.11}$$

In what follows, we shall derive the corresponding system of type IIB supergravity equations of motion describing radial evolution of the six unknown functions of u — x, y, z, w, f and Φ. We shall then discuss its solutions generalizing the work in [1] to the non-extremal case.

The simplest special fixed-point solution of our system turns out to have $w = 0$, i.e. the case when $T^{1,1}$ is not squashed. If the fractional branes are present, then the only U(1) symmetric solutions with $w = \Phi = 0$ are the KT solution [1] and its non-BPS generalization considered in [6]. We will discuss the latter solution in some detail in section 4, finding its *explicit* analytic form and clarifying its geometry. Its horizon turns out to be singular. A related problem is that it does not reduce to the regular non-extremal D3-brane solution in the limit of no fractional brane charge, $P \to 0$.

Demanding that the non-extremal solution have the correct $P = 0$ limit leads to the necessity of relaxing the condition $f' = -Pe^{\Phi+4y}$, i.e. that the 3-forms are self dual. If the 3-forms are not self-dual, then Φ and w can no longer be held constant. In the next section we derive the resulting system of second-order differential equations.

3. Basic equations

3.1 Effective 1-d action for radial evolution

As in [1], the most efficient way to derive the system of type IIB supergravity equations of motion is to follow [10, 11] and to start with the 1-d effective action for the radial evolution.

For the metric (2.1) $\sqrt{G} = \frac{1}{108} e^{10y-2z}$ (up to angle-dependent factors), and computing the scalar curvature we find

$$\int d^{10}x \sqrt{G} R \;\to\; \frac{1}{27}\int du [5y'^2 - 3x'^2 - 2z'^2 - 5w'^2 + e^{8y}(6e^{-2w} - e^{-12w})]\,. \tag{3.1}$$

Note that $w = 0$ is a consistent fixed point of the equations of motion. Replacing M_5 in (2.3) by the standard $T^{1,1}$ or by S^5 produces exactly the same 1-d gravitational lagrangian. That means, in particular, that *for* $w = 0$ the regular D3-brane solution and its *non-extremal* version will not change if we replace the flat transverse space $\mathbf{R}^6$ with the conifold.

It is easy to see that the matter part L_m of the effective lagrangian is essentially the same as it was in the extremal case [1] (apart from w-dependent factors): L_m does not depend on the non-extremality function x. Thus, following [1],

$$\int d^{10}x \sqrt{G}\, [-\frac{1}{2}(\partial\Phi)^2 + \cdots] \;\to \tag{3.2}$$
$$\to\; -\frac{1}{27}\int du\, \frac{1}{8}\Big[\Phi'^2 + 2e^{-\Phi+4z-4y-4w} f'^2 + 2e^{\Phi+4z+4y+4w} P^2 + e^{8z}(Q+2Pf)^2\Big]\,.$$

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From (3.1) and (3.2) we get the following 1-d effective lagrangian (ignoring an irrelevant overall numerical factor)

$$L = T - V\,, \tag{3.3}$$
$$T = 5y'^2 - 3x'^2 - 2z'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - \frac{1}{4}e^{-\Phi+4z-4y-4w}f'^2\,, \tag{3.4}$$
$$V = -e^{8y}(6e^{-2w} - e^{-12w}) + \frac{1}{4}e^{\Phi+4z+4y+4w}P^2 + \frac{1}{8}e^{8z}(Q+2Pf)^2\,, \tag{3.5}$$

supplemented with the "zero-energy" constraint $T + V = 0$.

Since the non-extremality function x does not appear in $\sqrt{G}$, G_{uu}, or the angular part of the metric, it is absent in L_m and thus is a "modulus" — it has no potential (cf. (3.1)). Thus its dependence on u is simply linear

$$x'' = 0\,, \qquad x = au\,, \qquad a = \text{const} > 0\,. \tag{3.6}$$

This is just what is expected of an extra kinetic energy (i.e. the $x'^2 = a^2$ term appearing in the "zero-energy" constraint) which spoils the BPS nature of the KT solution for a non-constant x.

3.2 The superpotential and the extremal KT solution

The crucial observation made in [1] is that the lagrangian (3.4), (3.5) has a remarkable special structure — it admits a superpotential. Indeed, it can be be represented in the following way

$$\begin{aligned} L = &-3x'^2 - \frac{1}{8}\Phi'^2 + \\ &+ 5\Big[y' + \frac{1}{5}e^{4y}(3e^{4w} + 2e^{-6w})\Big]^2 - 5\Big[w' - \frac{3}{5}e^{4y}(e^{4w} - e^{-6w})\Big]^2 - \\ &- 2\Big[z' + \frac{1}{4}e^{4z}(Q+2Pf)\Big]^2 \frac{1}{4}e^{-\Phi+4z-4y-4w}(f' + Pe^{\Phi+4y+4w})^2 - \\ &- 2\Big[\frac{1}{4}e^{4y}(3e^{4w} + 2e^{-6w}) - \frac{1}{8}e^{4z}(Q+2Pf)\Big]'\,, \end{aligned} \tag{3.7}$$

where the last term is a total derivative and may be dropped. Let us recall that, if, in general, $V(\phi)$ can be expressed in terms of a function $W(\phi)$ as $V = -g^{ij}\partial_i W \partial_j W$, where $g_{ij}(\phi)$ is the kinetic term metric, then

$$L = T - V = g_{ij}(\phi)\phi'^i\phi'^j - V(\phi) = g_{ij}(\phi'^i + g^{ik}\partial_k W)(\phi'^j + g^{jl}\partial_l W) - 2W'\,. \tag{3.8}$$

As a result, there exists a special BPS solution of the corresponding 2-nd order equations, satisfying

$$\phi'^i + g^{ik}\partial_k W = 0\,, \tag{3.9}$$

and thus also the zero-energy constraint. As follows from (3.7), in the present case [1]

$$W = \frac{1}{4}e^{4y}(3e^{4w} + 2e^{-6w}) - \frac{1}{8}e^{4z}(Q+2Pf)\,. \tag{3.10}$$

Note that W does not depend on Φ. The corresponding system (3.9) of 1-st order equations is then [1]:

$$x' = 0\,, \qquad \Phi' = 0\,, \tag{3.11}$$

$$y' + \frac{1}{5}e^{4y}(3e^{4w} + 2e^{-6w}) = 0\,, \qquad w' - \frac{3}{5}e^{4y}(e^{4w} - e^{-6w}) = 0\,, \tag{3.12}$$

$$f' + Pe^{\Phi+4y+4w} = 0\,, \qquad z' + \frac{1}{4}e^{4z}(Q + 2Pf) = 0\,. \tag{3.13}$$

The equation for f in (3.13) implies self-duality of the complex 3-form field, i.e. $H_3 = e^{\Phi} \star F_3$.

Choosing the special solution $w = 0$ of the w-equation in (3.12) we then find the KT solution [1]

$$\begin{aligned} x &= 0\,, \qquad w = 0\,, \qquad \Phi = 0\,, \\ e^{-4y} &= 4u\,, \qquad f = f_0 - \frac{P}{4}\ln u\,, \\ e^{-4z} &= 1 + K_0 u - \frac{P^2}{2}u(\ln u - 1)\,, \qquad K_0 = Q + 2Pf_0\,, \end{aligned} \tag{3.14}$$

i.e.

$$e^{-4z} = h = 1 + \left(Q + 2Pf_0 + \frac{P^2}{2}\right)u - \frac{P^2}{2}u\ln u\,. \tag{3.15}$$

In terms of the radial variable ρ used in [1],

$$\begin{aligned} e^{4y} &= \rho^4 = \frac{1}{4u}\,, \qquad f = f_1 + P\ln\rho\,, \\ h &= 1 + \left(Q + 2Pf_1 + \frac{P^2}{2}\right)\frac{1}{4\rho^4} + \frac{P^2}{2\rho^4}\ln\rho\,, \end{aligned} \tag{3.16}$$

where $f_1 = f_0 + \frac{P}{2}\ln 2$.

A more general extremal solution of (3.11)–(3.13) with non-zero w leads to fractional D3-branes on the generalized conifold (2.6) solution of [9].[3]

3.3 The full system of 2-nd order equations

We would like to generalize the solution (3.15) to the non-BPS case when $x' \neq 0$, i.e. the non-extremality parameter a in (3.6) is non-zero. In order to do that we need to start with the original 2-nd order system following from (3.4), (3.5) or (3.7), i.e. the

[3] For its explicit form in terms of the ρ coordinate, see [9, eqs. (4.20), (4.21)]. While the generalized conifold has, in contrast to the standard conifold, regular curvature, the back reaction of D3-branes makes the metric singular: pure D3-branes on the generalized conifold have a horizon coinciding with the singularity. The fractional D3-brane solution is similar to the KT one: it has a naked singularity behind the $K(u) = 0$ locus.

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free equation for x (3.6) plus a coupled system for y, w, z, f and Φ

$$10y'' - 8e^{8y}(6e^{-2w} - e^{-12w}) + \Phi'' = 0\,, \qquad (3.17)$$

$$10w'' - 12e^{8y}(e^{-2w} - e^{-12w}) - \Phi'' = 0\,, \qquad (3.18)$$

$$\Phi'' + e^{-\Phi+4z-4y-4w}(f'^2 - e^{2\Phi+8y+8w}P^2) = 0\,, \qquad (3.19)$$

$$4z'' - (Q+2Pf)^2 e^{8z} - e^{-\Phi+4z-4y-4w}(f'^2 + e^{2\Phi+8y+8w}P^2) = 0\,, \qquad (3.20)$$

$$(e^{-\Phi+4z-4y-4w}f')' - P(Q+2Pf)e^{8z} = 0\,. \qquad (3.21)$$

The integration constants are subject to the zero-energy constraint $T + V = 0$, i.e.

$$5y'^2 - 2z'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - \frac{1}{4}e^{-\Phi+4z-4y-4w}f'^2 -$$
$$- e^{8y}(6e^{-2w} - e^{-12w}) + \frac{1}{4}e^{\Phi+4z+4y+4w}P^2 + \frac{1}{8}e^{8z}(Q+2Pf)^2 = 3a^2\,. \qquad (3.22)$$

This system has special properties reflecting the structure of the lagrangian (3.7). In particular, there is a subclass of simple solutions for which the 1-st order equations for f and z (3.13) are still satisfied, while (3.11) and (3.12) are replaced by their 2-nd order counterparts. Indeed, it is easy to see that if we set $f'^2 - P^2 e^{2\Phi+8y+8w} = 0$, i.e. $f' = -Pe^{\Phi+4y+4w}$, then (3.21) implies that z should be subject to the first-order equation in (3.13). In this case the 3-forms are self-dual. Then it is consistent, in particular, to keep $w = 0$ so that $T^{1,1}$ is not squashed. We will discuss this class of solutions first in the next section.

In general, if we relax the 1-st order conditions (3.13) on f, z — and we will be forced to relax them in order to have a regular horizon — the dilaton will run according to (3.19), driven by the non-extremality $x' = a > 0$. From (3.18), if we relax the first order constraint on f, the function w will also be forced to run. Hence, we need the more general metric (2.1) in order to have a regular horizon. These nontrivial dilaton and w dynamics constitute a novel phenomenon specific to the non-extremal *fractional* D3-brane case, $a > 0$ and $P > 0$.

4. A singular non-BPS generalization of the KT solution

Assuming that f and z are subject to (3.13), i.e.

$$f' + Pe^{\Phi+4y+4w} = 0\,, \qquad z' + \frac{1}{4}e^{4z}(Q+2Pf) = 0\,, \qquad (4.1)$$

so that (3.20) and (3.21) are satisfied automatically, the remaining equations (3.17), (3.18), (3.19) and (3.22) become

$$5y'' - 4e^{8y}(6e^{-2w} - e^{-12w}) = 0\,, \qquad (4.2)$$

$$5w'' - 6e^{8y}(e^{-2w} - e^{-12w}) = 0\,, \qquad (4.3)$$

$$\Phi'' = 0\,, \qquad (4.4)$$

and

$$5y'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - e^{8y}(6e^{-2w} - e^{-12w}) = 3a^2 \,. \tag{4.5}$$

Notice that the matter and gravity parts are now decoupled: eqs. (4.2)–(4.5) would be obtained just by looking for Ricci-flat uncharged black 3-brane solutions with a metric in the class (2.1). Finding first the functions y and w, one is then to plug them back into (4.1) to determine the functions f and z.

The BPS solution of (4.2)–(4.5) corresponds to $a = 0$ and y, w subject to (3.12), leading to the generalized conifold space (2.6), (2.7).

We begin an analysis of the non-BPS solutions by discussing the special case when $w = \Phi = 0$. In this case, the $T^{1,1}$ is not squashed, and the eqs. (4.2)–(4.5) simplify substantially. When $P > 0$ we will see that we recover the non-BPS generalization of the KT solution first discussed in [6]. In section 5.2 we will see that in the $P = 0$ limit this solution does not become the standard (regular) non-extremal D3-brane solution. Similar singular non-BPS solutions with non-constant dilaton are constructed in appendix A.

Integrating (4.2) and using (4.5) we find

$$y' = -\sqrt{b^2 + e^{8y}} \,, \qquad b = \sqrt{\frac{3}{5}}a \,. \tag{4.6}$$

Assuming the required long-distance ($u \to 0$) asymptotic conditions we then have

$$e^{4y} = \frac{b}{\sinh 4bu} \,. \tag{4.7}$$

Integrating (4.1) shows that

$$f = f_* - \frac{P}{4}\ln\tanh 2bu \,, \tag{4.8}$$

i.e. f approaches a *constant* at large u and has the KT behavior (3.14) at small u.

The large u (short distance) behaviour is an improvement compared to the extremal KT case: since f stays positive (does not change sign) so does K which is a derivative of e^{-4z} (cf. (2.11), (4.1)). That means that e^{-4z} will keep growing at small distances, and does not go to zero. The total function $h = e^{-4z-4x}$ in (2.4) will vanish only at $u = \infty$ due to the vanishing of e^{-4x}, i.e. here the *singularity coincides with the horizon.*

This is a rather peculiar situation, different from the naked singularity of the KT solution. The non-vanishing of the non-extremality parameter $a \sim b$ is obviously crucial for this difference.[4]

[4]Other known cases of solutions where a horizon coincides with a curvature singularity include, for example, dilatonic BPS Dp-branes [12] as well as D3-branes on generalized conifolds [9].

Explicitly, from the equation for z in (4.1) $e^{-4z} = \int du\ [Q + 2Pf(u)]$, we find that z can be expressed in terms of polylogarithms

$$e^{-4z} = C + K_* u + \frac{P^2}{8b}\Big(\mathrm{Li}_2(-e^{-4bu}) - \mathrm{Li}_2(e^{-4bu})\Big)\,, \tag{4.9}$$

where $K_* = Q + 2Pf_*$.[5]

The exact analytical solution presented here realises the non-BPS KT background discussed in [6]. In particular, the equation (2.53) in [6] for the warp factor $\triangle_1(\tau) \equiv \sqrt{f(x)}$ that determines the position of the event horizon is solved with the identification[6]

$$x \equiv x(u) = \frac{4a}{3b} e^{-4au}\ \sinh 4bu\,, \tag{4.10}$$

$$f(x) \equiv f(x(u)) = e^{-8au}\,, \tag{4.11}$$

provided we set $a = \frac{243}{4}A$. In [6], $f(x_\star)$ was found numerically to vanish for $x_\star \neq 0$. Given (4.10), (4.11) we see that this statement is incorrect. This numerical error led to the wrong conclusion of the nonsingular horizon of the black hole solution in the KT geometry proposed in [6].

Near $u = 0$ we get from (4.9) the same expression as in the extremal case $(a = 0,\ b = 0)$, i.e. the KT behaviour (3.15) where e^{-4z} is 1 at $u = 0$ and grows as u increases. For large u:

$$y = -bu + \frac{1}{4}\log 2b + \frac{1}{4}e^{-8bu} + \cdots\,, \qquad f = f_* + \frac{P}{2}e^{-4bu} + \cdots\,, \tag{4.12}$$

$$e^{-4z} = C + K_* u - \frac{P^2}{4b}e^{-4bu} + \cdots\,. \tag{4.13}$$

Thus e^{-4z} always grows never reaching zero. If we define the horizon as the locus where $G_{00} = \exp(2z - 6x)$ vanishes, then from these large u asymptotics, it is clear that we have a horizon as $u \to \infty$. Moreover, from (2.1) and the differential equations (4.1), (4.2) and (4.5), we find that the Ricci scalar in this space-time is $R = P^2 e^{6z-6y}$. From the large u asymptotics, it is clear that R is singular in the limit $u \to \infty$.

We conclude that the non-BPS solution (4.7)–(4.9) does not have a horizon shielding the naked singularity of the extremal KT solution; instead, the introduction of non-extremality here creates a horizon at $u = \infty$ and shifts the singularity of the extremal KT background from a finite value of u to the same point $u = \infty$.

In fact, if we take the $P \to 0$ limit of (4.7)–(4.9), thus removing the fractional branes, we still have a singular horizon at $u = \infty$: even though the Ricci scalar vanishes, components of the curvature tensor blow up. The lesson is that the presence

[5]Note that $e^{-4z(0)} = C - \pi^2 P^2/32b$. The constant C may be adjusted so that the solution has or does not have an asymptotically flat region; the latter possibility corresponds to $h(0) = 0$.

[6]Here f and x refer to the notation in [6].

of the fractional charge does not influence the singularity significantly. We will show in the next section that this $P = 0$ limit corresponds *not* to the ordinary non-extremal D3-brane solution but to a special non-BPS D3-brane solution.

5. General non-extremal pure D3-solution and its regular and singular cases

In this section we trace the singular horizon problem of the special solution found in the previous section to a similar problem in the $P = 0$ case, i.e. to singularity of certain non-extremal generalizations of the regular extremal D3-brane solution.

In general, the system of second-order differential equations for y (3.18) and z (3.20) has an extra free integration constant — an extra parameter of non-BPS deformation in addition to the constant a in x in (3.6). The standard non-extremal D3 with regular horizon [12] is a special case of a more general class of solutions. One usually discards such more general solutions by imposing the condition that *the horizon should be regular.* That condition is satisfied only for a special choice of two free integration constants.

From the effective "7-d black hole" point of view, these more general solutions correspond to the case when an extra scalar (the radius of the internal 3-torus) has non-vanishing asymptotic charge. However, regular black holes should not have scalar hair — otherwise we get a singular horizon. It is only when this extra scalar charge is tuned to zero that we get a regular non-extremal D3-brane solution.

To consider the non-extremal pure D3-brane case let us start with the general system of equations (3.17)–(3.22) with $P = 0$ and $f = 0$.[7] To match the standard D3-brane solution we shall also set $\Phi = 0$ and $w = 0$.[8] Then we are left with (3.6) and the following system:

$$y'' - 4e^{8y} = 0\,, \qquad z'' - \frac{1}{4}Q^2 e^{8z} = 0\,, \tag{5.1}$$

i.e.

$$x' = a\,, \qquad y'^2 = b^2 + e^{8y}\,, \qquad z'^2 = c^2 + q^2 e^{8z}\,, \qquad q \equiv \frac{1}{4}Q\,, \tag{5.2}$$

with the integration constants a, b, c related by the zero-energy constraint (3.22)

$$5b^2 - 3a^2 - 2c^2 = 0\,. \tag{5.3}$$

Assuming that $a, b, c \geq 0$ (so that $y \to \infty$ for $u \to 0$) and that h, g in (2.5) approach 1 as $u \to 0$, we find

$$e^{4y} = \frac{b}{\sinh 4bu}\,, \qquad e^{4z} = \frac{c}{q\ \sinh 4c(u+k)}\,, \qquad e^{4x} = e^{4au}\,, \tag{5.4}$$

[7] We could keep f non-constant for $P = 0$. That would introduce an extra potential term in the equations for the remaining fields. Like the $\Phi \sim u$ case discussed in appendix A, this corresponds to having an extra scalar charge and most likely leads to a singular solution.

[8] The discussion of this section applies both to $M_5 = T^{1,1}$ and $M_5 = S^5$.

where k is defined by

$$e^{4ck} = q^{-1}(\sqrt{q^2+c^2}+c) \equiv \gamma\,. \tag{5.5}$$

Then (see (2.5))

$$\rho^4 = e^{4y+4x} = \frac{2be^{4(a-b)u}}{1-e^{-8bu}}\,, \qquad g = e^{-8au}\,, \tag{5.6}$$

$$h = e^{-4z-4x} = e^{4(c-a)u}\Big[1+\frac{q}{2c\gamma}(1-e^{-8cu})\Big]\,. \tag{5.7}$$

At small u (large ρ) we have

$$g = 1-\frac{2a}{\rho^4}+\cdots\,, \qquad h = 1+\frac{\sqrt{q^2+c^2}-a}{\rho^4}+\cdots\,, \qquad \rho^4 = \frac{1}{4u}+\cdots\,. \tag{5.8}$$

5.1 Standard regular non-extremal D3-brane solution

The *standard* non-extremal D3-brane solution [12] corresponds to the case when

$$b = c = a\,, \tag{5.9}$$

i.e. to a line in the 2-parameter (b,c) space. Then the constraint (5.3) is satisfied, and (5.4), (5.5) become

$$e^{4y} = \frac{a}{\sinh 4au}\,, \qquad e^{4z} = \frac{a}{q\sinh 4a(u+k)}\,, \qquad e^{4x} = e^{4au}\,, \tag{5.10}$$

$$\gamma = e^{4ak} = q^{-1}(\sqrt{q^2+a^2}+a)\,. \tag{5.11}$$

Note that near the horizon ($u \to \infty$)

$$y = y_* - au + y_1 e^{-8au} + O(e^{-16au})\,, \qquad z = z_* - au + z_1 e^{-8au} + O(e^{-16au})\,, \tag{5.12}$$

$$y_* = \frac{1}{4}\ln 2a\,, \qquad y_1 = \frac{1}{4}\,, \qquad z_* = \frac{1}{4}\ln\frac{2a}{q\gamma}\,, \qquad z_1 = \frac{1}{4\gamma^2}\,, \tag{5.13}$$

while at large distances ($u \to 0$)

$$y = -\frac{1}{4}\ln 4u - \frac{2}{3}a^2u^2 + O(u^3)\,, \qquad z = -\bar{q}u + O(u^2)\,, \qquad \bar{q} = q\gamma^{-1}+a = \sqrt{q^2+a^2}\,. \tag{5.14}$$

It is only for the choice $b = c = a$ that the metric (2.4) takes the standard non-extremal D3-brane form [12, 13]

$$ds^2_{10E} = h^{-1/2}(gdX_0^2 + dX_idX_i) + h^{1/2}[g^{-1}d\rho^2 + \rho^2(dM_5)^2]\,, \tag{5.15}$$

$$g = e^{-8x} = 1-\frac{2a}{\rho^4}\,, \qquad \rho^4 = \frac{2a}{1-e^{-8au}}\,, \tag{5.16}$$

$$h = e^{-4z-4x} = 1+\frac{\tilde{q}}{\rho^4}\,, \qquad \tilde{q} = \gamma^{-1}q = \sqrt{q^2+a^2}-a\,. \tag{5.17}$$

In an often-used parametrization $q = 2a\sinh\alpha\cosh\alpha$ and $\tilde{q} = 2a\sinh^2\alpha$, where the charge q is fixed in the extremal $a \to 0$ limit. The mass (density) of the solution is $M = \sqrt{q^2 + a^2} + \frac{3}{4}a \ > \ q$, so this is the standard non-extremal black-brane type solution with a *regular* horizon.

Let us note also that the choice of $k = 0$ in (5.10) leads to the black hole in AdS solution, where we remove the asymptotically flat region. Then the metric is (5.15) with

$$g = e^{-8x} = 1 - \frac{2a}{\rho^4}, \qquad \rho^4 = \frac{2a}{1 - e^{-8au}}, \qquad h = e^{-4z-4x} = \frac{q}{\rho^4}. \tag{5.18}$$

5.2 Special singular non-extremal D3-brane solution

The general solution with arbitrary b and c reduces to the standard extremal D3-brane background only if we set b and c proportional to a, satisfying the constraint (5.3).

The simplest special case is $c = 0$ where z satisfies the 1-st order equation $z' = -qe^{4z}$ (cf. (5.2)). Then from (5.4) we get

$$e^{4y} = \frac{b}{\sinh 4bu}, \qquad e^{-4z} = 1 + 4qu\,, e^{4x} = e^{4au}, \qquad b = \sqrt{\frac{3}{5}}a\,, \tag{5.19}$$

and thus

$$\rho^4 = e^{4y+4x} = \frac{2be^{4(a-b)u}}{1 - e^{-8bu}}, \qquad h = e^{-4z-4x} = (1 + 4qu)e^{-4au}, \qquad g = e^{-8au}. \tag{5.20}$$

This solution, which is the $P = 0$ *limit* of the solution [6] derived in section 4, has a *singular* horizon at $u = \infty$.

For small u (large distances) and in the limit $a \to 0$, we still get the standard asymptotic extremal D3-brane behavior

$$\rho^4 = \frac{1}{4u} + \cdots, \qquad g = 1 - \frac{2a}{\rho^4} + \cdots, \qquad h = 1 + \frac{q-a}{\rho^4} + \cdots. \tag{5.21}$$

If we define the horizon as the place where G_{00} vanishes, then its location is at infinite u. Note that there $g \to 0$, but also $h \to 0$; still $G_{00} = h^{-1/2} g \to 0$.

Since $b < a$, the radial coordinate ρ is not a monotonic function of u: it grows to infinity at both $u = 0$ and $u = \infty$, having a minimum at finite u. Therefore, ρ is not a good (one-to-one) coordinate, and we must instead use u to cover the entire space-time. The same remark applies to the fractional brane generalization of (5.20), i.e. to the solution of [6] found in section 4.

It is only in the case $b = c = a$ of the *standard* D3-brane solution (5.10) that ρ is monotonic with $\rho(0) = \infty$, and $\rho^4(\infty) = 2a$ is the position of the horizon.

The explicit form of the corresponding metric (2.4) in the special case of (5.20) is

$$ds^2_{10E} = (1 + 4qu)^{-1/2}(e^{-6au}dX_0^2 + e^{2au}dX_i dX_i) + (1 + 4qu)^{1/2} ds_6^2\,, \tag{5.22}$$

$$ds_6^2 = \left(\frac{b}{\sinh 4bu}\right)^{5/2} du^2 + \left(\frac{b}{\sinh 4bu}\right)^{1/2} (dM_5)^2\,. \tag{5.23}$$

This metric has a horizon as well as curvature singularity at $u = \infty$. (One can check that $R_{mnkl}R^{mnkl}$ is divergent there.)

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6. Discussion

To summarize, the condition of regular horizon usually imposed on black holes excludes the special non-extremal D3-brane solution discussed in section 5.2. This solution is special in that z satisfies the same 1-st order equation (without an extra integration constant) as in the extremal case. For that reason it is this solution that has the immediate simple fractional D3-brane generalization found in [6] and described explicitly in section 4. This solution has a singular horizon, which is now not surprising since for zero fractional D3-brane charge P, it reduces *not* to the standard regular D3-brane solution of section 5.1 but to the special singular solution of section 5.2.

Let us make the comparison with [6] more explicit. Just as for the $P = 0$ case discussed in section 5.2, as we increase u the radial coordinate ρ reaches a minimum value ρ_* and then starts increasing again. In fact, $\rho(u)$ is exactly the same as in the $P = 0$ case, eq. (5.20). In [6] ρ_* was incorrectly identified as the horizon. But it is not a horizon because $g = e^{-8x}$ is finite there — the minimum value ρ_* is an artefact of the coordinate choice. In this solution the coordinate u covers both branches of the ρ coordinate. The singular horizon is located at $u = \infty$ where $\rho = \infty$.

The above discussion shows that in order for the non-extremal generalization of the KT solution to reduce to the standard black D3-brane in the $P \to 0$ limit, the large u asymptotics have to be (see (5.10), (5.12))

$$u \gg 1: \qquad x = au\,, \qquad y \to -au + y_*\,, \qquad z \to -au + z_*\,, \tag{6.1}$$

$$w \to w_*\,, \qquad \Phi \to \Phi_*\,, \qquad f \to f_*\,. \tag{6.2}$$

These asymptotics guarantee the existence of a regular Schwarzschild horizon at $u = \infty$, and it is natural to expect that w, f and Φ have stationary points at this $u \to \infty$ horizon. Then it is easy to see that our system of equations (3.17)–(3.21) *and* the constraint (3.22) are indeed satisfied at large u. It is also not hard to check that turning on P makes a small perturbation on these asymptotics.

Since in the $P \to 0$ limit we need to keep $c = a$ in the equation (5.2) for z, we are forced to give up the 1-st order equations for z and f (4.1), i.e. the self-duality of the 3-forms. In turn, the equations (3.19) and (3.18) for Φ and w then receive sources and it is no longer possible to have solutions with $\Phi = 0$ and $w = 0$. The reason for this special role of w is that our ansatz for the forms (2.8)–(2.10) breaks the symmetry between ψ and other directions of M_5. The lack of self-duality of the 3-forms causes the $T^{1,1}$ to become squashed at finite u.

Now we face a formidable task of solving the complete 2-nd order system of equations (3.17)–(3.22), looking for special non-singular solutions which interpolate between the KT asymptotics (3.14), (3.15) at small u and the Schwarzschild horizon asymptotics (6.1), (6.2) for large u. Such solutions will be discussed in a future publication.

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A. Non-BPS solution with non-constant dilaton and $w = 0$

Here we discuss singular solutions with a non-constant dilaton,

$$\Phi = 4pu\,. \tag{A.1}$$

Now the zero-energy constraint is

$$5y'^2 - 5w'^2 - e^{8y}(6e^{-2w} - e^{-12w}) = 3a^2 + 2p^2\,. \tag{A.2}$$

We find

$$e^{4y} = \frac{b}{\sinh 4bu}\,, \qquad 5b^2 = 3a^2 + 2p^2\,. \tag{A.3}$$

The functions f and z are then determined from (4.1).

If we keep a and p independent and take $P = 0$, then we do not get the standard non-extremal D3-brane solution but rather its generalization with non-zero dilatonic charge. If $a = 0$ but the non-vanishing dilaton charge $p \neq 0$, we recover the conifold analog of the singular generalization of the extremal D3-brane solution discussed in [14, 15]. The fractional 3-brane case with non-zero P still leads to a singular solution.

It is therefore necessary to relate p and a, i.e. to demand that

$$p = ma\,, \qquad b = na\,, \qquad n = \left[\frac{1}{5}(3 + 2m^2)\right]^{1/2}. \tag{A.4}$$

The relation (A.4) is needed in order to get back the KT solution (and not its other non-BPS generalization with running dilaton) in the limit $a = 0$. Then a plays the role of the non-extremality parameter which "drives" the solution away from the BPS point of the KT background (3.15).

One particularly simple possibility is $m = 1$, i.e.

$$p = b = a\,. \tag{A.5}$$

Then eqs. (4.1) become

$$f' + \frac{aPe^{4au}}{\sinh 4au} = 0\,, \qquad (e^{-4z})' = Q + 2Pf\,, \tag{A.6}$$

so that

$$f = f_* - \frac{P}{4}\ln(e^{8au} - 1)\,, \tag{A.7}$$

which of course reduces to the KT expression in (3.14) in the BPS $a = 0$ limit ($f_0 = f_* - \frac{P}{4}\ln 8a$). f goes in the same $\log u$ way for small u (large distances) and linearly with u for large u (small distances), a novel behaviour.

The equation for z gives

$$\begin{aligned} e^{-4z} &= C + K_* u - 2aP^2u^2 - \frac{P^2}{16a}\mathrm{Li}_2(e^{-8au})\,, \\ C &= 1 + \frac{P^2\pi^2}{96a}\,, \qquad K_* = Q + 2f_*P\,, \end{aligned} \tag{A.8}$$

where C is such that $z(0) = 0$ to have the standard long distance limit. Explicitly, for small u we reproduce the KT asymptotic behaviour

$$e^{-4z} = 1 - \frac{P^2}{2}u\log u + \Big[K_* + \frac{1}{2}P^2(1 - \ln 8a)\Big]u + O(u^2)\,, \tag{A.9}$$

while for large u

$$e^{-4z} = -2aP^2u^2 + K_* u + O(e^{-8au})\,, \tag{A.10}$$

indicating the presence of a special point at finite u where e^{-4z} vanishes.[9]

This point is a curvature singularity. According to (2.4), (2.5), $h = e^{-4z-4x}$, so that $h = 0$ at *finite* u. Note that $g = e^{-8x}$ and $\rho = e^{y+x}$ are still finite there, so this is a *naked singularity*. Just as in the KT case, the derivative of z or $(e^{-4z})' = K(u)$ becomes zero at $u = u_0$ before we reach that singular point: $e^{8au_0} = 1 + 2P^{-1}e^{K_0}$. Since the derivative of e^{-4z} changes sign, that means e^{-4z} reaches a maximum at $u = u_0$ and then goes to zero.

Above we considered the case of $p > 0$ when the string coupling $e^\Phi = e^{4pu}$ grows at small distances. One finds a somewhat nicer behaviour of the metric in the opposite case of $m = -1$, i.e.

$$p = -b = -a\,. \tag{A.11}$$

Now for large u the string coupling becomes weak and $f \to f_*$. This produces a solution with a singular horizon at $u = \infty$. For example, if we choose $Q + 2Pf_* = 0$, then e^{-4z} approaches a constant for large u. The horizon is singular because the string-frame metric becomes

$$ds^2_{\rm string} \to e^{-8au}dX_0^2 + dX_i dX_i + e^{-12au}du^2 + e^{-4au}(dM_5)^2\,,$$

so that the longitudinal volume stays finite but the transverse volume vanishes at the horizon.

[9]Numerical analysis confirms that there exists a finite value of u were z goes to infinity. The behaviour of e^{-4z} is similar to that of the $1 + u - u^2$ function: its starts at 1, grows reaching a maximum, and then goes to zero at approximately $u_s = 1.617$.

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